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Algebraic method for parameter identification of circuit models for batteries under non-zero initial condition☆

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HIGHLIGHTS

- Algebraic method derived for identification of circuit models for batteries.
- Zero initial condition of capacitor voltage not required.
- Rest time can be substantially reduced between subsequent tests.
- Battery modeling and evaluation can be accelerated.

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ABSTRACT

This paper presents an algebraic method for parameter identification of Thevenin's equivalent circuit models for batteries under non-zero initial condition. In traditional methods, it was assumed that all capacitor voltages have zero initial conditions at the beginning of each charging/discharging test. This would require a long rest time between two tests, leading to very lengthy tests for a charging/discharging cycle. In this paper, we propose an algebraic method which can extract the circuit parameters together with initial conditions. This would theoretically reduce the rest time to 0 and substantially accelerate the testing cycles.

1. Introduction

Several types of battery models are used to evaluate the performances of batteries and to study their interactions with other electrical devices. As compared with electrochemical and mathematical models, the electric circuit based models are much simpler for computation and analysis. They have a high potential in terms of accuracy and parameterization effort [1]. They are the most intuitive for use in circuit simulations [2] and can be easily integrated with other electric devices and control algorithms at a system level [6].

A widely used circuit based models for batteries are Thevenin's equivalent models as depicted in Fig. 1.

It was initially proposed in Ref. [24] in 1992 for led-acid batteries and then widely used in Refs. [8–34] to model various types of batteries such as lead-acid, lithium-ion (Li-ion), Li-polymer, nickel metal hydride (NiMH), and fuel cells.

Thevenin's equivalent models have been used for analysis, design, and simulation of battery powered electronic systems [8,10,11,16–18,20,33]. They are also important for characterization of battery performance, life-time estimation, power management, and efficient use of batteries [21–25]. In Ref. [1], three Thevenin's equivalent circuit models were applied to simulate a real-life driving cycle of an electric vehicle. In Ref. [4], the models were used for online estimation of state-of-charge and open circuit voltage of lithium-ion batteries in electric vehicles.

The circuit model in Fig. 1 has several variations. For example, the ideal voltage source can be replaced with a capacitor for describing the charging dynamics or longer time discharging behavior. The ideal voltage source can also be replaced with another pair of capacitor and resistor in parallel to account for the

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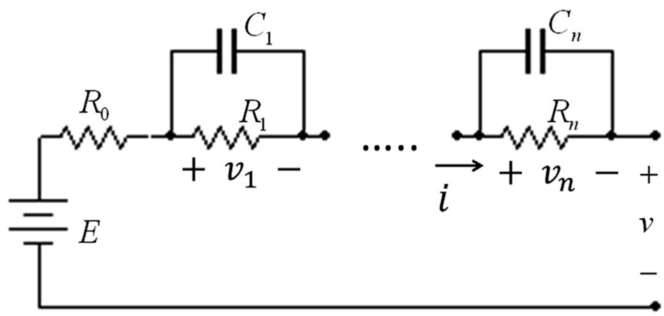

Fig. 1. Thevenin's equivalent circuit model for batteries.

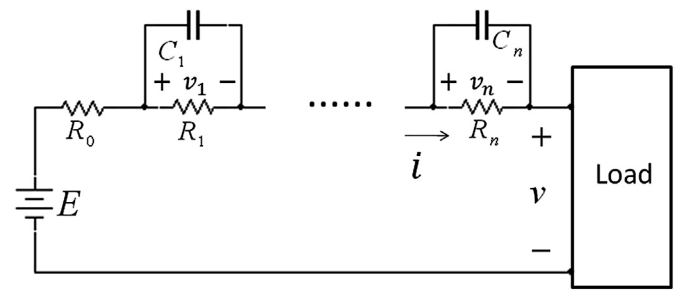

Fig. 2. A battery connected to an electronic load.

self-discharging behavior. These variations have also been widely considered in the literature (e.g., see Refs. [9,24]). More advanced circuit models have been developed based on Fig. 1. For example, in Ref. [35], the independent voltage source is replaced by a controlled voltage source which is dependent on the voltage of a capacitor in parallel with a resistor and a dependent current source.

It is known that the parameters in the circuit model depend on many factors, such as state of charge (SOC), the load current, the temperature and even the history of charge and discharge [8,20,24,26]. They can be regarded as constants under a certain working condition and over a relatively short period of time. More complex nonlinear models have been proposed based on the relationship between the circuit parameters and the operating conditions [11,14,15,20]. In Ref. [5], the resistances R_0 and R_2 were dependent on the open circuit voltage and the capacitor voltage.

A core task in battery modeling is to identify the parameters in the Thevenin's equivalent circuit model via some experimental responses under a given operating condition. With families of parameters identified for various working conditions, a nonlinear model describing the dependence of these parameters on SOC, current, and temperature, can be obtained with numerical methods [14]. In Refs. [2,5], multi-objective genetic algorithms were developed for extracting the parameters of the linear/nonlinear circuit models.

The circuit parameters are traditionally identified by using least-square curve fitting of the experiment data. In Refs. [12,13], we developed simple analytical methods for identifying the parameters for several equivalent circuit models including the one in Fig. 1.

A complete nonlinear or parameter dependent circuit model for a battery requires many rounds of charging or discharging cycles. Each round consists of many tests for the characterization of the parameters' dependence on SOC. Between two subsequent tests, a sufficient rest time is required due to the assumption of zero initial condition at the beginning of each tests. The rest time could be 30 min or longer, depending on the type of battery [36].

In this paper, we propose an algebraic method for extracting the circuit parameters together with the initial conditions. The assumption of non-zero initial condition will eliminate the need for rest time or reduce the rest time, thereby accelerate the modeling process.

2. Algebraic method for extracting the parameters and the initial conditions

We will first show that it is impossible to extract the parameters and the initial conditions at the same time when a constant load current is applied. If the load current is a step function, i.e., it takes one value over a period of time and then switch to another value, then the parameters and the initial conditions can be identified simultaneously.

2.1. Parameter identification via a constant load current

For simplicity and without loss of generality, we consider the battery model with 2 pairs of parallel resistors and capacitors, (R_1,C_1),(R_2,C_2), as depicted in Fig. 1. This will be called the 2nd-order model. The method can be easily extended to 3rd or higher order by using the algebraic method in Ref. [12].

A common setup to identify the parameters is to connect the battery to an electronic load which absorbs a prescribed current i from the battery, see Fig. 2.

The current can be a constant, a step function, a square wave, or other types of programmable functions. The terminal voltage $v(t)$ is recorded and used for parameter identification.

In this section, we consider a constant load current $i = I$ and assume that the load is connected at $t = 0$ (implying $i = 0$ for $t < 0$). A traditional assumption made on the circuit is that the initial conditions $v_1(0)$,$v_2(0)$,…, are all 0. In this paper, we don't make this assumption and allow the initial conditions to be nonzero and unknown. Denote

$$v_1(0) = v_{10}, \quad v_2(0) = v_{20}.$$

The instantaneous terminal voltage before applying the current load is

$$v(0^-) = E - v_{10} - v_{20} \tag{1}$$

Under a constant current $i = I$, the terminal voltage response for $t > 0$ is

$$v(t) = E - R_0I - R_1I - R_2I - (v_{10} - R_1I)e^{-\frac{t}{R_1C_1}} - (v_{20} - R_2I)e^{-\frac{t}{R_2C_2}} \tag{2}$$

Comparing (1) and (2), we have $R_0 = (v(0^-)-v(0^+))/I$.

The other parameters, E, R_1,R_2,C_1,C_2, and initial condition v_{10},v_{20} need to be computed from the voltage response $v(t)$. Recall that under the assumption of zero initial condition $v_{10} = 0$,$v_{20} = 0$, all the parameters can be identified from the terminal voltage response using various methods. However, when v_{10},v_{20} are nonzero and also unknown, the parameters cannot be identified based on the response in (2), in spite of obtaining infinitely many equations for only 7 unknowns. This is because the parameters cannot be separated from the initial condition under a constant discharging current. This fact is formally stated in the following claim.

Claim 1. Let the terminal voltage $v(t)$ be given for $t > 0$. The variables E, R_1,R_2,C_1,C_2,v_{10},v_{20} cannot be uniquely determined from $v(t)$. Denote $s_1 = v_{10}-R_1I$, $s_2 = v_{20}-R_2I$ and $\tau_1 = R_1C_1$, $\tau_2 = R_2C_2$.

1. s_1,s_2,τ_1,τ_2 can be uniquely determined from 5 points of $v(t)$.
2. For the 5 unknowns, E,R_1,R_2,v_{10},v_{20}, there are only three linearly independent equations.

A constructive proof for the claim is given below. We first show item 1. Denote $s_1 = v_{10}-R_1I$, $s_2 = v_{20}-R_2I$. Choose $T > 0$ and let $d_1 = e^{-T/R_1C_1}, d_2 = e^{-T/R_2C_2}$. Then for any integer $k \geq 0$,

$$v(kT) = E - R_0I - R_1I - R_2I - s_1d_1^k - s_2d_2^k,$$

The constant term $E-R_0I-R_1I-R_2I$ can be eliminated by subtracting $v((k+1)T)$ from $v(kT)$. To be specific,

$$v(0^+) - v(T) = s_1(d_1-1) + s_2(d_2-1)$$

$$v(T) - v(2T) = s_1(d_1-1)d_1 + s_2(d_2-1)d_2$$

$$v(2T) - v(3T) = s_1(d_1-1)d_1^2 + s_2(d_2-1)d_2^2$$

$$v(3T) - v(4T) = s_1(d_1-1)d_1^3 + s_2(d_2-1)d_2^3$$

Define $x_1 = s_1(d_1-1)$, $x_2 = s_2(d_2-1)$, the above equations can be rewritten as

$$x_1 + x_2 = v\left(0^+\right) - v(T)$$

$$x_1d_1 + x_2d_2 = v(T) - v(2T)$$

$$x_1d_1^2 + x_2d_2^2 = v(2T) - v(3T)$$

$$x_1d_1^3 + x_2d_2^3 = v(3T) - v(4T)$$

From the above equations, the 4 variables x_1,x_2,d_1,d_2 can be easily solved by using the algebraic method in [12] (details will be provided in Section 2.3). Then s_1,s_2,τ_1,τ_2 can be computed from these variables:

$$s_1 = \frac{x_1}{d_1-1}, \quad s_2 = \frac{x_2}{d_2-1},$$

$$\tau_1 = R_1C_1 = -\frac{T}{\ln d_1}, \quad \tau_2 = R_1C_2 = -\frac{T}{\ln d_1}.$$

With s_1,s_2,τ_1,τ_2 uniquely determined, we have

$$E - R_1I - R_2I = v(t) + R_0I + s_1e^{-t/\tau_1} + s_2e^{-t/\tau_2} \tag{3}$$

The right-hand-side of the above equation must be a constant since I is a constant. Thus for the 5 unknowns E,R_1,R_2,v_{10},v_{20}, there are only 3 linearly independent equations, namely, (3) and $v_{10}-R_1I = s_1$, $v_{20}-R_2I = s_2$. Therefore, these variables cannot be uniquely determined.

By Claim 1, we know that the circuit parameters can not be determined with unknown initial conditions because there are not sufficient linearly independent equations for the unknown variables.

2.2. Parameter identification via a step load current

A simple approach to introduce more linearly independent equations is to change the value of the current to a different constant. To be specific, we choose a switching time T_0 and let

$$i(t) = \begin{cases} I, & 0<t<T_0 \\ I_1, & t>T_0 \end{cases}$$

We can use the measurement from 0 to T_0 to find s_1,s_2,τ_1,τ_2 and use the measurement after T_0 to form two more linearly independent equations.

Claim 2. All the circuit parameters and the initial conditions can be identified from measurement taken by applying a step load current.

A constructive proof for this claim is given below.

Choose $T < T_0/4$. We can use the method in the previous section to compute $s_1 = v_{10}-R_1I$, $s_2 = v_{20}-R_2I$ and $\tau_1 = R_1C_1$, $\tau_2 = R_2C_2$ by using $v(t)$ for $t < T_0$.

Denote $\alpha_1 = e^{-T_0/\tau_1}, \ \alpha_2 = e^{-T_0/\tau_2}$. Then at T_0, the two capacitor voltages are

$$v_1(T_0) = s_1\alpha_1 + R_1I,$$

$$v_2(T_0) = s_2\alpha_2 + R_2I.$$

Choose $T_1 > 0$. Let $p_1 = e^{-T_1/\tau_1}, \ p_2 = e^{-T_1/\tau_2}$. Then for $k = 0,1,2,\ldots$,

$$\begin{aligned} v_1(T_0+kT_1) &= (v_1(T_0) - R_1I_1)p_1^k + R_1I \\ &= (s_1\alpha_1 + R_1(I-I_1))p_1^k + R_1I \\ v_2(T_0+kT_1) &= (v_2(T_0) - R_2I_1)p_2^k + R_2I \\ &= (s_2\alpha_2 + R_2(I-I_1))p_2^k + R_2I \end{aligned}$$

Denote

$$\begin{aligned} y_1 &= s_1\alpha_1 + R_1(I-I_1) \\ y_2 &= s_2\alpha_2 + R_2(I-I_1). \end{aligned} \tag{4}$$

We have

$$v(T_0+kT_1) = E - R_0I_1 - R_1I_1 - R_2I_1 - y_1p_1^k - y_2p_2^k$$

By subtracting two subsequent points, we obtain

$$v(T_0^+) - v(T_0+T_1) = y_1(p_1-1) + y_2(p_2-1)$$

$$v(T_0+T_1) - v(T_0+2T_1) = y_1(p_1-1)p_1 + y_2(p_2-1)p_2$$

Since p_1,p_2 are given, we can find y_1, y_2 from the above two equations. Since s_1,s_2,α_1,α_2 are given, we can compute R_1,R_2 from (4). Then $C_1 = \tau_1/R_1$, $C_2 = \tau_2/R_2$ and $v_{10} = s_1+R_1I$, $v_{20} = s_2+R_2I$. Finally, we can compute parameter E from (2).

2.3. Algorithms for parameter identification

For completeness, we provide the algorithms by summarizing the arguments and steps from the previous sections and using the method in Ref. [12].

Let the load current be I for $0 < t < T_0$ and I_1 for $t < T_0$, respectively. The terminal voltage $v(t)$ is recorded at $t = 0^-, \ t = 0^+$ and for $t > 0$. Then $R_0 = (v(0^-)-v(0^+))/I$.

Algorithm for 2nd-order model. Choose $T \in (0,T_0/4]$ and $T_1 > 0$. To extract the 7 parameters E,R_1,R_2,C_1,C_2 and v_{10},v_{20}, we will need

the measurement of the terminal voltage $v(t)$ at 8 time instants: $v(0^+)$,$v(T)$,$v(2T)$, $v(3T)$, $v(4T)$, $v(T_0^+)$, $v(T_0+T_1)$, $v(T_0+2T_1)$.

Step 1: Compute $s_1 = v_{10}-R_1I$, $s_2 = v_{20}-R_2I$ and $\tau_1 = R_1C_1$, $\tau_2 = R_2C_2$.

For $k = 1,2,3,4$, let $b_k = v((k-1)T)-v(kT)$.

Compute $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} b_2 & -b_1 \\ b_3 & -b_2 \end{bmatrix}^{-1} \begin{bmatrix} b_3 \\ b_4 \end{bmatrix}$.

Let the roots to the second order equation $d^2-u_1d+u_2 = 0$ be d_1,d_2. Then

$$\tau_1 = -\frac{T}{\ln(d_1)}, \quad \tau_2 = -\frac{T}{\ln(d_2)}$$

Let $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ d_1 & d_2 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Then

$$s_1 = \frac{x_1}{d_1 - 1}, \quad s_2 = \frac{x_2}{d_2 - 1}$$

Step 2: Use $v(T_0^+)$, $v(T_0 + T_1)$, $v(T_0 + 2T_1)$ to complete the computation.

Let $\alpha_1 = e^{-T_0/\tau_1}, \alpha_2 = e^{-T_0/\tau_2}, p_1 = e^{-T_1/\tau_1}, p_2 = e^{-T_1/\tau_2}$.
Compute

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} p_1 - 1 & p_2 - 1 \\ p_1(p_1 - 1) & p_2(p_2 - 1) \end{bmatrix}^{-1} \begin{bmatrix} v\left(T_0^+\right) - v(T_0 + T_1) \\ v(T_0 + T_1) - v(T_0 + 2T_1) \end{bmatrix}$$

Then

$$R_1 = \frac{y_1 - s_1\alpha_1}{I - I_1}, \quad R_2 = \frac{y_2 - s_2\alpha_2}{I - I_1},$$

$$C_1 = \tau_1/R_1, \quad C_2 = \tau_2/R_2$$

Finally,

$$v_{10} = s_1 + R_1I, \quad v_{20} = s_2 + R_2I,$$

and $E = v(0^-) + v_{10} + v_{20}$.

For 3rd or higher order models, the algorithm can be extended from the arguments in previous sections and by using the algebraic tool developed in [12].

Algorithm for 3rd-order model. Choose $T \in (0, T_0/6]$ and $T_1 > 0$. To extract the 10 parameters E,R_1,R_2,R_3,C_1,C_2,C_3 and v_{10},v_{20},v_{30}, we will need the measurement of the terminal voltage $v(t)$ at 11 time instants: $v(0^+)$,$v(kT)$,$k = 1,2,\ldots,6$, $v(T_0^+)$, $v(T_0+kT_1)$, $k = 1,2,3$.

Step 1: Compute $s_j = v_{j0}-R_jI$ and $\tau_j = R_jC_j$, $j = 1,2,3$ from $v(0^+)$,$v(kT)$,$k = 1,\ldots,6$.

For $k = 1,\ldots,6$, let $b_k = v((k-1)T)-v(kT)$.
Compute

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} b_3 & -b_2 & b_1 \\ b_4 & -b_3 & b_2 \\ b_5 & -b_4 & b_3 \end{bmatrix}^{-1} \begin{bmatrix} b_4 \\ b_5 \\ b_6 \end{bmatrix}$$

Let the roots to $q^3 - 2u_1q^2 + (u_1^2 + u_2)q + (u_3 - u_1u_2) = 0$ be q_1,q_2,q_3. Let

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Then

$$\tau_j = -\frac{T}{\ln(d_j)}, \ j = 1, 2, 3$$

Let

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ d_1 & d_2 & d_3 \\ d_1^2 & d_2^2 & d_3^2 \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Then

$$s_j = \frac{x_j}{d_j - 1}, \ j = 1, 2, 3$$

Step 2: Let $\alpha_j = e^{-T_0/\tau_j}, \ p_j = e^{-T_1/\tau_j}, \ j = 1, 2, 3$.

$$\text{Form } Q = \begin{bmatrix} p_1 - 1 & p_2 - 1 & p_3 - 1 \\ p_1(p_1 - 1) & p_2(p_2 - 1) & p_3(p_3 - 1) \\ p_1^2(p_1 - 1) & p_2^2(p_2 - 1) & p_3^2(p_3 - 1) \end{bmatrix}.$$

Compute

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = Q^{-1} \begin{bmatrix} v\left(T_0^+\right) - v(T_0 + T_1) \\ v(T_0 + T_1) - v(T_0 + 2T_1) \\ v(T_0 + 2T_1) - v(T_0 + 3T_1) \end{bmatrix}$$

Then

$$R_j = \frac{y_j - s_j\alpha_j}{I - I_1}, \quad C_j = \tau_j/R_j, \quad j = 1, 2, 3$$

Finally,

$$v_{j0} = s_j + R_jI, \quad j = 1, 2, 3,$$

and

$$E = v\left(0^-\right) + v_{10} + v_{20} + v_{30}.$$

2.4. Validation of the algorithms and sensitivity issues

The algorithms can be easily implemented with Matlab. Four Matlab codes are included in the Appendix for verification of the algorithms for a second-order model and a third-order model, with specific explanations for the notation and steps. For each model, one Matlab code is used to generate and plot the voltage response at specified time instants, given all the parameters and initial conditions. Another Matlab code uses the voltage response to reconstruct all the parameters and initial conditions by using the algorithm.

Both algorithms were validated except for very small numerical errors. Here we use the computational results for the 3rd-order model to demonstrate the key algebraic steps. There are three 3 by 3 matrices in the Algorithm which need to take inverse. Their eigenvalues are given below:

$$eig\left(\begin{bmatrix} b_3 & -b_2 & b_1 \\ b_4 & -b_3 & b_2 \\ b_5 & -b_4 & b_3 \end{bmatrix}\right) = \{0.0486, -0.0217, -0.0009\}$$

$$eig\left(\begin{bmatrix} 1 & 1 & 1 \\ d_1 & d_2 & d_3 \\ d_1^2 & d_2^2 & d_3^2 \end{bmatrix}\right) = \{1.1960, 0.6844, 0.0964\}$$

$$eig(Q) = \{-0.9864, -0.2682, -0.0392\}$$

The original parameters and the reconstructed are compared as follows

	Original	Reconstructed
R_1	0.060000000000000	0.060000000000630
R_2	0.050000000000000	0.050000000000336
R_3	0.040000000000000	0.039999999998466
C_1	200.000000000000	199.999999998905
C_2	640.000000000000	640.000000007105
C_3	2800.000000000000	2800.000000131836
v_{10}	0.010000000000000	0.010000000000088
v_{20}	0.020000000000000	0.020000000000599
v_{30}	−0.050000000000000	−0.050000000002005

The average relative error (defined as $|1-\text{reconstructed/original}|$) is 1.98×10^{-11}.

For the second-order model, the average relative error is much smaller, 7.08×10^{-13}.

For a real voltage response recorded in the experiment, measurement error and noises are unavoidable. Hence it is necessary to examine the sensitivity of the reconstructed parameters with respect to voltage perturbation. If all voltage values are increased (or decreased) at the same time, the reconstructed parameters do not change much. The parameters are more sensitive to the type of perturbation where the voltage is increased at one instant but decreased the next. So we used the following perturbation pattern: $v(0^+)+\delta$, $v(T)-\delta$,$v(2T)+\delta$, …, $v(T_0^+) - \delta, v(T_0 + T_1) + \delta, \ldots$

Fig. 3 shows some relationship between the average relative parameter deviation and the perturbation δ. Clearly, the third-order model is much more sensitive than the second-order model. On the curve for the 3rd-order model, there is a non-smooth point, which occurs at $\delta = 1.56 \times 10^{-5}$. At this perturbation value, the reconstructed C_1 is a complex number, which is useless. For the 2nd-order model, complex capacitance will also occur, but at a much larger δ: $\delta = 5.03 \times 10^{-4}$.

Due to these sensitivity issues, when the algorithm for a 3rd-order model is used on experimental responses, the reconstructed circuit parameters usually have complex numbers. When the algorithm for the 2nd-order model is applied, the circuit parameters are usually real numbers and we can always adjust the time interval T and T_1 to produce reasonable parameters with small root-mean-square error between experimental response and model response.

2.5. Considerations for extracting circuit parameters from experimental responses

If the voltage response $v(t)$ is computed from an ideal circuit model, then all the circuit parameters can be reconstructed, with small numerical errors, from several points: $v(0^-)$,$v(0^+)$,$v(kT)$,$v(T_0+kT_1)$,$k = 1,2,\ldots$ The results will not be affected significantly by the choices of T and T_1, except for very small numerical errors. However, the experimental responses have various nonidealities, such as measurement errors and quantization errors. Furthermore, the current is not a strict step function and has jitters or small ripple. After all, the circuit model is only an approximation of the complex chemical processes inside a battery.

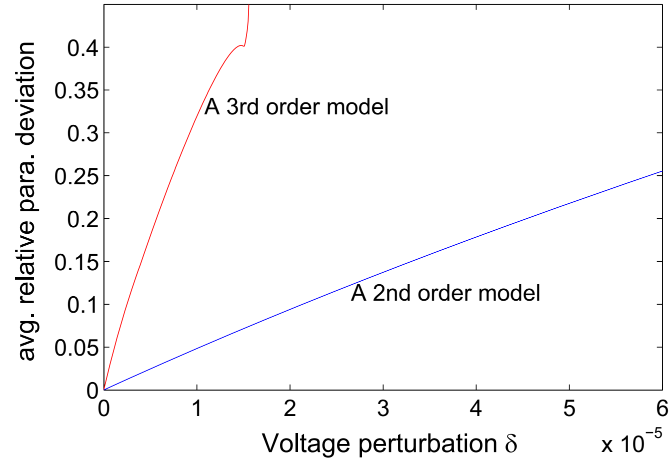

Fig. 3. Sensitivity to voltage perturbation.

Because of all these nonidealities, the parameters extracted from the experimental responses will depend on T, T_1 and T_0. For some values of T and T_1, the algorithm may produce negative or even complex numbers for the resistance and capacitance. This kind of situation would be encountered more frequently when higher-order models are extracted. Thus we need to vary T and T_1 within an admissible range and choose the resulting circuit parameters which produce the minimal root-mean-square-error (RMSE) between the experimental response and the response reconstructed from the model.

For simplicity, we may set $T_1 = T$ and use one dimensional sweep to find the best value T.

Another consideration is the choice of the difference between load currents I and I_1. Since the parameters depend on the discharging current, the difference should be kept as small as possible. However, with very small differences, the computed parameters may be too sensitive to noises and measurement errors and the results would vary widely from test to test. In our experiment, different values of $I-I_1$ were tested and a suitable one was chosen.

The total test time and the switch time T_0 depend on what kind of dynamic properties are evaluated and the capacity of the battery. For brief transient properties, a few seconds or minutes of response may be sufficient; for long time discharging/charging under steady load current, hours of test time may be required.

3. Experiment and computational results on a lead-acid battery

The experiment was conducted to demonstrate the use of our algorithm in identifying circuit parameters under non-zero initial condition of the capacitor voltage. It was not intended for the complete evaluation of a battery under all operating conditions. Thus the tests were only run unde some particular operating conditions, such as a specific discharging current, state of charge and temperature. Although the parameters under different operating conditions can be used together to derive more complex nonlinear models, this is not in the scope of this paper.

The battery used for the experiment was a lead acid battery rated 6V and 12 Ah. The 2nd-order Thevenin's equivalent circuit model was considered. We examined how the circuit parameters depended on the discharging current under a certain state of charge.

A group of tests were conducted under room temperature (25 °C). For each test, the current load was programmed to draw a step current which was initially I and then switched to $I_1 = I + 0.2A$ at T_0. The value of I varies from 0.6 A to 1.8 A. Each test lasted about 2 min. The rest time between two subsequent tests was between 1 and 5 min. In earlier similar experiment for the work [12], the rest time was about 30 min.

The terminal voltage responses under the step current were recorded via a 16-digit data acquisition (DAQ) device. The input voltage range was −10 V to 10 V. Thus the resolution was $20/2^{16} = 3.0518^{-4}$ V. No digital or analog filter was used to process the data/signals.

With the test data, we computed parameters for a 2nd-order model under different discharging current.

Fig. 4 shows the terminal voltage $v(t)$ for a step discharging current with $I = 0.6A$ for $t \in (0,75)$ and $I_1 = 0.8A$ for $t > 75$ s. The experimental response is plotted in pink (light colored) curve. The black curve is the response reconstructed from the circuit model whose parameters were extracted from the experimental response by our proposed algorithm.

For simplicity, we chose the same value for T and T_1. The best T was obtained via a one dimensional sweep within a certain range so that the RMSE is minimal.

The computational result gave $T = T_1 = 3.69$ s, $RMSE = 0.0008$ V. The extracted parameters and initial conditions are given below:

$$R_0 = 0.0267\Omega,\ R_1 = 0.0258\Omega,\ R_2 = 0.0412\Omega$$

$$C_1 = 53.79F,\ C_2 = 474.3F$$

$$v_{10} = -0.0011V,\quad v_{20} = -0.0157V,\quad E = 6.1548V$$

If zero initial conditions were assumed, we obtained different parameters (except for R_0), as given below,

$$R_1 = 0.0289\Omega,\ R_2 = 0.0668\Omega$$

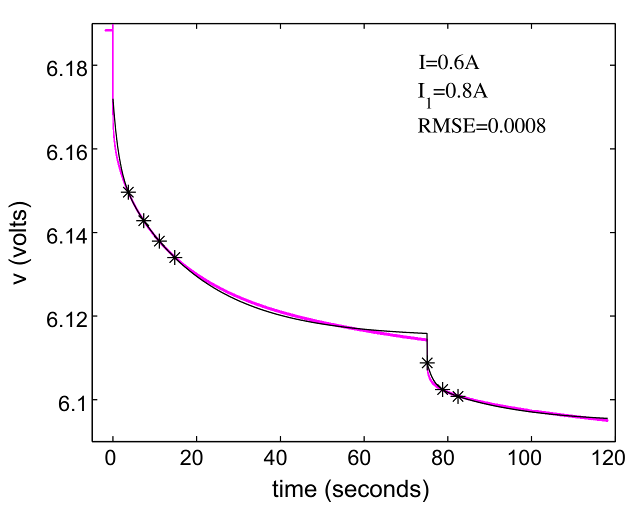

Fig. 4. Discharging responses: $I = 0.6A$, $I_1 = 0.8A$.

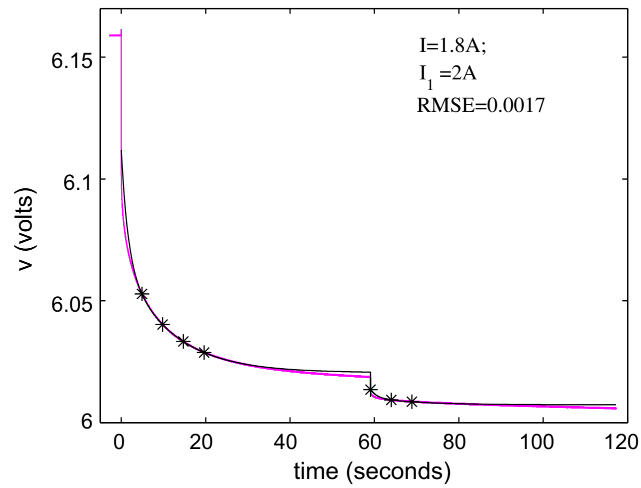

Fig. 5. Discharging responses: $I = 1.8A$, $I_1 = 2A$.

$$C_1 = 50.9F,\ C_2 = 308.1F$$

Fig. 5 shows the experimental response for $I = 1.8A$ and $I_1 = 2A$ with switching time $T_0 = 59.09$ s. The computational result gives $T = T_1 = 4.905$ s, $RMSE = 0.0017$. The larger RMSE is due to larger discharging current and larger voltage drop. The parameters and initial conditions are given below:

$$R_0 = 0.0275\Omega,\ R_1 = 0.0166\Omega,\ R_2 = 0.0133\Omega,$$

$$C_1 = 94.9734F,\ C_2 = 854.7153F$$

$$v_{10} = -0.0149V,\quad v_{20} = -0.0227V,\quad E = 6.1107V$$

When zero initial conditions are assumed, we obtained the following parameters,

$$R_1 = 0.0246\Omega,\ R_2 = 0.0262\Omega$$

$$C_1 = 61.44F,\quad C_2 = 428.9F$$

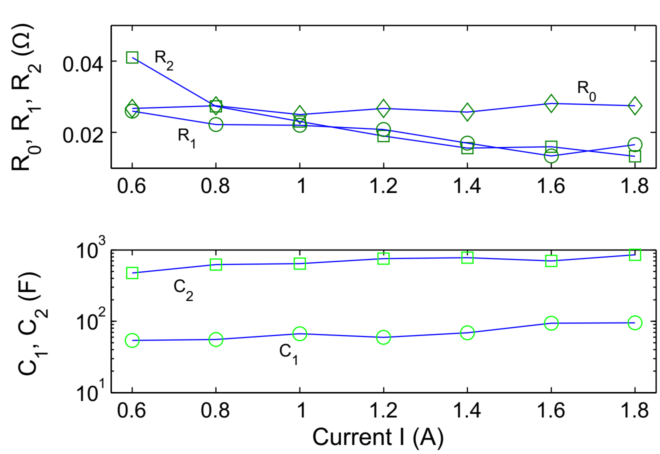

Fig. 6. Parameters vs discharging current.

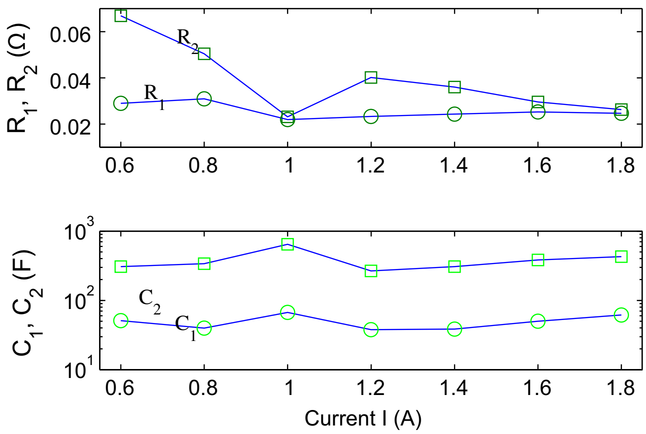

Fig. 7. Parameters vs discharging current, 0 initial condition assumed.

We obtained experimental responses for $I = 0.6,0.8,1.0,1.2,1.4,1.8A$ and $I_1 = I+0.2A$. Circuit parameters were computed for each current. Fig. 6 plots the computed circuit parameters vs the discharging current I. There are visible changes in R_2. Other parameters remain nearly flat. The curves show a clear pattern of the parameters and good consistency among the group of tests. The curves are similar to those in Fig. 11 in [12], which were obtained for a different lead-acid battery.

If zero initial conditions were assumed, different parameters would be produced. Fig. 7 show the result when zero initial conditions are assumed for the computation.

There are more visible changes on the curves and the patterns of the curves are not as clear as those in Fig. 6. These numbers may lead to more complex relationship and functions for the dependence of the parameters on the current.

4. Conclusions

This paper derived an algebraic method for identifying parameters for circuit models of batteries with unknown non-zero initial conditions. It was shown that such a parameter identification problem can not be solved with a constant load current and that a step load current is necessary. Since the new method does not require zero initial condition, the rest time between two subsequent tests can be substantially reduced, thus accelerating the testing cycles for battery evaluation.

A. Matlab codes

A.1. Matlab codes for a 2nd-order model

A.1.1. Matlab code for generating voltage response under step current

```
%Circuit parameters. R1C1,R2C2 in ascending order
E=6;
R0=0.02;
C1=200;R1=0.06;


C2=2800;R2=0.04;


%Initial conditions
v10=0.02;v20=-0.05;


%Values for step current and time intervals
I=2;I1=2.2; T0=200; %I for t<T0, I1 for t>T0
T=30;T1=30;


%exponential constants: d_i,alpha_i,p_i, i=1,2
dm1=exp(-T/R1/C1);af1=exp(-T0/R1/C1);p1=exp(-T1/R1/C1);
dm2=exp(-T/R2/C2);af2=exp(-T0/R2/C2);p2=exp(-T1/R2/C2);
```

```
v0=E-v10-v20-R0*I; %stands for v(0+)
s=E-(R0+R1+R2)*I;
v1=s-(v10-R1*I)*dm1-(v20-R2*I)*dm2; % for v(T)
v2=s-(v10-R1*I)*dm1^2-(v20-R2*I)*dm2^2;
v3=s-(v10-R1*I)*dm1^3-(v20-R2*I)*dm2^3;
v4=s-(v10-R1*I)*dm1^4-(v20-R2*I)*dm2^4; %for v(4T)

vT0=s-(v10-R1*I)*af1-(v20-R2*I)*af2; %for v(T0-)
z1=(v10-R1*I)*af1+R1*(I-I1);
z2=(v20-R2*I)*af2+R2*(I-I1);
s1=E-(R0+R1+R2)*I1;
vT00=s1-z1-z2; %for v(T0+)
vT01=s1-z1*p1-z2*p2; %for v(T0+T1)
vT02=s1-z1*p1^2-z2*p2^2; %for v(T0+2T1)
tt=[0 0 T T*2 T*3 T*4 T0 T0 T0+T1 T0+T1*2];
vv=[E-v10-v20 v0 v1 v2 v3 v4 vT0 vT00 vT01 vT02];
%plot(tt,vv) %for plotting the voltage response

%adding perturbations
z=0  %for no perturbation
v0=v0+z;v1=v1-z;

             v2=v2+z;v3=v3-z;
             v4=v4+z;vT00=vT00-z;
             vT01=vT01+z;vT02=vT02-z;
```

A.1.2. Matlab code for reconstructing parameters and initial condition

```
% Given values of voltage response at specified time instants,
% step discharging current I, I1, with stepping time T0,
% T and T1 are time intervals before/after T0

%Step 1: Computing d1,d2,s1,s2
   b1=v0-v1; b2=v1-v2;         %v0 for v(0+)
   b3=v2-v3;  b4=v3-v4;        %vn for v(nT)
   u=inv([b2 -b1;b3 -b2])*[b3;b4];

   d=roots([1 -u(1) u(2)]);
   d1=min(d);d2=max(d);

   x=inv([1 1; d1 d2])*[b1;b2];
   s1=x(1)/(d1-1);s2=x(2)/(d2-1);

%Step 2: Reconstructing parameters
   p1=d1^(T1/T); p2=d2^(T1/T);
   af1=d1^(T0/T); af2=d2^(T0/T);
   A=[p1-1 p2-1;p1*(p1-1) p2*(p2-1)];
   B=[vT00-vT01;vT01-vT02];
   y=inv(A)*B;

% Reconstruct R1, R2
   Re1=(y(1)-s1*af1)/(I-I1);
   Re2=(y(2)-s2*af2)/(I-I1);

%Reconstruct C1, C2
   Ce1=-T/Re1/log(d1);
   Ce2=-T/Re2/log(d2);
```

```
%Reconstruct v10, v20, E;
   ve10=Re1*I-x(1)/(1-d1);
   ve20=Re2*I-x(2)/(1-d2);
   E=v0+R0*I+ve10+ve20;

%Relative error between given parameters and reconstructed one
% R0 excluded since it only depends on v(0-) and v(0+)
   ee=[1-ve10/v10;1-ve20/v20;1-Re1/R1;1-Re2/R2;1-Ce1/C1;1-Ce2/C2;1-E/6];
%average of relative error
   avgee=sum(abs(ee))/7;
```

A.2. Matlab codes for a 3rd-order model

A.2.1. Generating voltage response under step current

```
%Circuit parameters. R1C1,R2C2,R3C3 in ascending order
   E=6;
   R0=0.02;
   C1=200;R1=0.06;
   C2=640;R2=0.05;
   C3=2800;R3=0.04;

%Initial conditions
   v10=0.01;v20=0.02;v30=-0.05;

%Values for step current and time intervals
   I=2;I1=2.2; T0=200; %I for t<T0, I1 for t>T0
   T=30;T1=30;

%exponential constants: d_i,alpha_i,p_i, i=1,2,3
   dm1=exp(-T/R1/C1);af1=exp(-T0/R1/C1);p1=exp(-T1/R1/C1);
   dm2=exp(-T/R2/C2);af2=exp(-T0/R2/C2);p2=exp(-T1/R2/C2);
   dm3=exp(-T/R3/C3);af3=exp(-T0/R3/C3);p3=exp(-T1/R3/C3);

%Computing voltage response at 0+,T,2T,...,T0,T0+T1,T0+2T1,...
```

```
    v=zeros(1,12);
    s=E-(R0+R1+R2+R3)*I;
    for n=1:7  %v(n) stands for v((n-1)T)
      v(n)=s-(v10-R1*I)*dm1^(n-1)-(v20-R2*I)*dm2^(n-1)-(v30-R3*I)*dm3^(n-1);
    end

    v(8)=s-(v10-R1*I)*af1-(v20-R2*I)*af2-(v30-R3*I)*af3; %v(8) for v(T0-);
    z1=(v10-R1*I)*af1+R1*(I-I1);
    z2=(v20-R2*I)*af2+R2*(I-I1);
    z3=(v30-R3*I)*af3+R3*(I-I1);
    s1=E-(R0+R1+R2+R3)*I1;
    v(9)=s1-z1-z2-z3; %v(T0+)
    v(10)=s1-z1*p1-z2*p2-z3*p3; %v(T0+T1)
    v(11)=s1-z1*p1^2-z2*p2^2-z3*p3^2; %v(T0+2T1)
    v(12)=s1-z1*p1^3-z2*p2^3-z3*p3^3; %v(T0+3T1)

% Plotting the voltage response
    tt=[0 0 T T*2 T*3 T*4 T*5 T*6 T0 T0 T0+T1 T0+T1*2 T0+T1*3];
    vv=[E-v10-v20-v30 v];  %v(0-)=E-v10-v20-v30
%plot(tt,vv,'*') % The voltage response can be plotted

%Add some perturbation to the voltage response
    ve=[1 -1 1 -1 1 -1 1 -1 1 -1 1 -1];
    delta=0; %Choose a small number for delta
    v=v+ve*delta;
% The vector v contains all the voltage values at 11 specified points
% They will be used to reproduce the circuit parameters along with
% I, I1, T0, T, T1
```

A.2.2. Matlab code for reconstructing parameters and initial condition

```
    % Given values of voltage response at specified time instants,
    % step dischaging current I, I1, with stepping time T0,
    % T and T1 are time intervals before/after T0
    % Three pairs of parallel RC
```

```
%Step 1: Computing d1,d2,d3,s1,s2,s3
   b1=v(1)-v(2); b2=v(2)-v(3);   %v(1) stands for v(0+)
   b3=v(3)-v(4);  b4=v(4)-v(5);  %v(n) for v((n-1)T);
   b5=v(5)-v(6); b6=v(6)-v(7);
   u=inv([b3 -b2 b1;b4 -b3 b2;b5 -b4 b3])*[b4;b5;b6];

   q=roots([1 -2*u(1) u(1)^2+u(2) u(3)-u(1)*u(2)]);
   d=inv([0 1 1;1 0 1;1 1 0])*q;
   d=sort(d); %line up d1 d2 d3 in ascending order
   d1=d(1);d2=d(2);d3=d(3);

   x=inv([1 1 1; d1 d2 d3;d1^2 d2^2 d3^2])*[b1;b2;b3];
   s1=x(1)/(d1-1);s2=x(2)/(d2-1);s3=x(3)/(d3-1);

%Step 2: Computing circuit parameter and initial conditions
   p1=d1^(T1/T); p2=d2^(T1/T); p3=d3^(T1/T);
   af1=d1^(T0/T); af2=d2^(T0/T); af3=d3^(T0/T);
   A=[p1-1 p2-1 p3-1;
      p1*(p1-1) p2*(p2-1) p3*(p3-1);
      p1^2*(p1-1) p2^2*(p2-1) p3^2*(p3-1)];
   B=[v(9)-v(10);v(10)-v(11);v(11)-v(12)];
 %v(9) for v(T0+); v(10) for v(T0+T1)
   y=inv(A)*B;

%Reconstruct R1, R2, R3
   Re1=(y(1)-s1*af1)/(I-I1);
   Re2=(y(2)-s2*af2)/(I-I1);
   Re3=(y(3)-s3*af3)/(I-I1);

% Reconstruct C1, C2, C3
   Ce1=-T/Re1/log(d1);  %T/log(d1) for tau1
   Ce2=-T/Re2/log(d2);
   Ce3=-T/Re3/log(d3);
```

```
%Reconstruct initial conditions v10, v20, v30 and E
   ve10=Re1*I-x(1)/(1-d1);
   ve20=Re2*I-x(2)/(1-d2);
   ve30=Re3*I-x(3)/(1-d3);
   E=v(1)+R0*I+ve10+ve20+ve30;

%Relative error between given parameters and reconstructed one
% R0 excluded since it only depends on v(0-) and v(0+)
   ee=[1-ve10/v10;1-ve20/v20;1-ve30/v30;1-E/6;
    1-Re1/R1;1-Re2/R2;1-Re3/R3;
    1-Ce1/C1;1-Ce2/C2;1-Ce3/C3];
%average of relative deviation
   avgee=sum(abs(ee))/10;
```

References

[1] M. Einhorn, F.V. Conte, C. Kral, J. Fleig, IEEE Trans. Power Electron. 28 (3) (2013) 1429–1437.
[2] J. Brand, Z. Zhang, R.K. Agarwal, J. Power Sources 247 (2014) 729–737.
[4] H. He, X. Zhang, R. Xiong, Y. Xu, H. Guo, Energy 39 (1) (2012) 310–318.
[5] A. Malik, Z. Zhang, R.K. Agarwal, J. Power Sources 259 (2014) 76–86.
[6] K.S. Hariharan, V.S. Kumar, J. Power Sources 222 (2013) 210–217.
[8] M. Ceraolo, IEEE Trans. Power Syst. 15 (4) (2000) 1184–1190.
[9] M.A. Cassaca, Z.M. Salameh, IEEE Trans. Energy Convers. 7 (3) (1992) 442–444.
[10] S.X. Chen, K.J. Tseng, S.S. Choi, Modeling of lithium-ion battery for energy storage system simulation, in: Power and Energy Engineering Conference, 2009, pp. 1–4.
[11] L. Gao, S. Liu, R.A. Dougal, IEEE Trans. Components Packag. Tech. 25 (3) (2002) 495–505.
[12] T. Hu, B. Zanchi, J. Zhao, IEEE Trans. Energy Convers. 26 (3) (2011) 787–798.
[13] T. Hu, H. Jung, J. Power Sources (2013, 233, 14-22).
[14] Y. Hu, S. Yurkovich, J. Power Sources 196 (5) (2011) 2913–2923.
[15] T. Kim, W. Qiao, IEEE Trans. Energy Convers. 26 (4) (Dec. 2011) 1172–1180.
[16] R.C. Kroeze, P.T. Krein, Electrical battery model for use in dynamic electric vehicle simulations, in: IEEE Conference on Power Electronics Specialists, 2008, pp. 1336–1342.
[17] Palma, M. Harfman-Todorovic, P. Enjeti, S. Choi, Analysis of DC-DC converter stability in fuel cell powered portable electronic systems, in: 37th IEEE Power Electronics Specialists Conference, June, 2006, pp. 1–6.
[18] C. Speltino, D. Di Domenico, G. Fiengo, A. Stefanopoulou, Comparison of reduced order lithium-ion battery models for control applications, in: Joint 48th IEEE Conf. on Deci. and Contr. and 28th Chinese Contr. Conf., 2009, pp. 3276–3281.
[19] K. Sun, Q. Shu, Overview of the types of battery models, in: Proc. of the 30th Chinese Control Conference (CCC), July 2011, pp. 3644–3648.
[20] S. Tian, M. Hong, M. Ouyang, IEEE Trans. Energy Convers. 24 (2) (2009) 452–458.
[21] L. Benini, G. Castelli, A. Macii, E. Macii, M. Poncino, R. Scarsi, A discrete-time battery model for high-level power estimation, in: Proc. of the Conf. on Design, Automat. and Test in Europe, 2000, pp. 35–41.
[22] M. Durr, A. Cruden, S. Gair, J.R. McDonald, J. Power Sources 161 (2) (2006) 1400–1411.
[23] J.F.A. Leao, L.V. Hartmann, M.B.R. Correa, A.M.N. Lima, Lead-acid battery modeling and SOC monitoring, in: Applied Power Electronics Conference and Exposition (APEC), 2010, pp. 239–243.
[24] Z.M. Salameh, M.A. Casacca, W.A. Lynch, IEEE Trans. Energy Convers. 7 (1) (1992) 93–98.
[25] J. Zhang, C. Song, H. Sharif, M. Alahmad, An enhanced circuit-based model for single-cell battery, in: Applied Power Electronics Conference and Exposition, 2010 Twenty-fifth Annual IEEE, 2010, pp. 672–675.
[26] S. Barsali, M. Ceraolo, IEEE Trans. Energy Convers. 17 (1) (2002) 16–23.
[27] H.-C. Chang, C.-M. Liaw, IEEE Trans. Power Electron. 23 (4) (2008) 2143–2156.
[28] N. Jantharamin, L. Zhang, A new dynamic model for lead-acid batteries, in: IET Conf. on Power Electronics, Machines and Drives, 2008, pp. 86–90.
[29] G. Livint, M. Ratoi, V. Horga, M. Albu, Estimation of battery parameters based on continuous-time model, in: Int. Sym. on Signals, Circuits and Systems, vol. 2, 2007, pp. 1–4.
[30] D.D.-C. Lu, V.G. Agelidis, IEEE Trans. Power Electron. 24 (3) (2009) 849–855.
[31] N. Moubayed, J. Kouta, A. El-Ali, H. Dernayka, R. Outbib, Parameter identification of the lead-acid battery model, in: Photovoltaic Specialists Conference, 2008, pp. 1–6.
[32] C. Wang, M.H. Nehrir, S.R. Shaw, IEEE Trans. Energy Convers. 20 (2) (2005) 442–451.
[33] Y. Yao, F. Fassinou, T. Hu, IEEE Trans. Power Electron. 26 (9) (Sept. 2011) 2614–2626.
[34] J. Zhang, C. Song, H. Sharif, M. Alahmad, IEEE Trans. Energy Convers. 25 (4) (2010) 1133–1141.
[35] M. Chen, G.A. Rincon-Mora, IEEE Trans. Energy Convers. 21 (2) (2006) 504–511.
[36] M.A. Roscher, D.U. Sauer, J. Power Sources (Jan, 2011) 331–336.